Recap - Nov 7, 2018

Definition. *The ringed space* $(X, \mathcal{O}_X)$ *is a locally ringed space if for each* $x \in X$*, the stalk* $\mathcal{O}_{X,x}$ *is a local ring. We define the residue field of* x *to be the field* $k(x) := \mathcal{O}_{X,x}/m_{X,x}$*, where* $m_{X,x}$ *is the maximal ideal of* $\mathcal{O}_{X,x}$*.*

Example. $X = Spec\ R$ *is a locally ringed space.* $\mathcal{O}_{X,\mathfrak{p}} = R_{\mathfrak{p}}$*.*

Definition. *A morphism of ringed spaces from* $(Y, \mathcal{O}_Y)$ *to* $(X, \mathcal{O}_X)$ *is a pair* $(\pi, \pi^{\#})$ *of a continuous map* $\pi : Y \to X$ *and a map* $\pi^{\#} : \mathcal{O}_X \to \pi_* \mathcal{O}_Y$ *of sheaves of rings on* Y *such that for each* $x \in X$ *the induced map of local rings* $\pi_x^{\#} : \mathcal{O}_{X,\pi(y)} \to \mathcal{O}_{Y,y}$ *is a local homomorphism of local rings.*

Proposition. *Let* R *be a ring,* $X = Spec\ R$ *and* Y *be a pre-variety. Then given a map of pre-schemes* $\pi : Y \to X$ *is the same as giving a compatible ring homomorphism* $R \to \Gamma(U, \mathcal{O}_Y)$*.*

Corollary.

$$\{\textit{affine schemes}\} \cong \{\textit{rings}\}$$

$$Hom(Spec\ S, Spec\ R) = Hom(R, S)$$

Definition. *Let* Z *be a fixed pre-scheme. A pre-scheme over* Z *is a pre-scheme* X*, together with a morphism* $X \to S$*. If* X *and* Y *are pre-schemes over* S*, a morphism of* X *to* Y *as pre-schemes over* S *is a morphism which is compatible with the given morphisms to* S*.*

Corollary. *Every pre-scheme live over Spec* Z*.*

Let $U \subset X$. Try to define

$$\Gamma(U, \mathcal{O}_X) = \{\varphi : U \to \coprod_{p \in U} k(p) \text{ with } \varphi(p) \in k(p) \text{ and } U = \cup X_{g_\alpha}, s_\alpha \in R_{g_\alpha},$$

$$\varphi(Q) = \text{ image of } s_\alpha \text{ in } k(Q) \ \ \forall s_\alpha \in Q\}$$

Exercise. *Show this works if* R *is reduced.*