AG Daily recap

October 10, 2018

Definition. $\pi : Y \to X$ is an affine map if there exists an open cover $X = \bigcup U_i$ such that $U_i \subseteq X$ is open affine so that $\pi^{-1}(U_i) = V_i$ is affine.

Theorem. $\pi : Y \to X$ is affine if and only if for all open affine subset $U \subseteq X$ we have $V := \pi^{-1}(U)$ affine.

For example,

- the map from $\mathbb{C}^2 \setminus \{(0,0)\}$ to $\mathbb{C}^2$ is not affine;
- Closed immersion is affine;
- Any finite map is affine.

Lemma. $\pi : Y \to X$ finite implies π is closed and quasi finite. (This was an exercise from last time) Note that we can assume π to be dominant.

Proposition. Let $\pi : Y \to X$ be dominant, then $\pi(Y)$ contains a nonempty open subset.

Definition. A contructible set = finite union of locally closed set.

Corollary (Chevalley Theorem.) For any regular map $\pi : Y \to X$,

$$\pi(\text{constructible set}) = \text{another constructible set}$$

Exercise. Prove the proposition above implies the Chevalley theorem.
Note that

- It suffices to check $\pi(Y)$ constructible;
- May assume π is dominant;
- Now break things into smaller dimensional pieces.