Math 552, Algebraic Geometry
October 19, 2018 Lecture Recap

Recall the following theorem from the preceding lecture.

Theorem 1. *Let $\pi : Y \to X$ be a dominant morphism of algebraic varieties.*

1. *For $u \in X$ and $Z \subseteq \pi^{-1}(\{u\}) \subseteq Y$ an irreducible component of the fiber of π over u, we have* $\dim(Z) \geq \dim(Y) - \dim(X)$.
2. *There is a nonempty open subset $U \subseteq X$ such that for $u \in U$ and Z as above,* $\dim(Z) = \dim(Y) - \dim(X)$.
3. *The function $y \mapsto \dim_y(\pi^{-1}(\pi(y)))$ is upper semi-continuous as a function of $y \in Y$.*

We actually proved a slight strengthening of the second point: (in the same setting) there exists a nonempty open subset $U \subseteq X$ such that for all irreducible closed subsets $W \subseteq X$ such that $W \cap U \neq \emptyset$, any component of $\pi^{-1}(W)$ dominating W has dimension $\dim(Y) - \dim(X) + \dim(W)$ (such a component always exists, as W is irreducible).

Corollary 2. *For $\pi : Y \to X$ a morphism of varieties, and $W \subseteq X$ an irreducible closed subset such that all of the components of the fibers $f^{-1}(\{w\})$ are of the same dimension, for all $w \in W$, then $f^{-1}(W)$ is irreducible.*