Daily Recap

September 21, 2018

Definition. *A morphism* $\pi : Y \to X$ *is a continuous map of topological space such that the pullback inducing* $\mathcal{O}_X \to \pi_* \mathcal{O}_Y$*, i.e. for all open sets* $U \subset X$*,* $\varphi \in \Gamma(U, \mathcal{O}_X) \Rightarrow \varphi \circ \pi \in \Gamma(\pi^{-1}(U), \mathcal{O}_Y)$*.*

Note. *A function being regular is a local property.*

Let $X \subset \mathbb{P}_k^n$ be a projective variety.

Proposition. $\pi : X \to \mathbb{P}_k^m$ *is regular if and only if it is locally given by* $m + 1$ *homogeneous polynomials of the same degree.*

Example (Twisted Cubic)**.** *Define* $\pi : \mathbb{P}^1 \to \mathbb{P}^3$ *by* $[s : t] \mapsto [s^3 : s^2 t : st^2 : t^3]$*.*

Example. $X = \mathbb{V}(ad - bc) \subset \mathbb{P}^3$*. Define* $\pi_1 : X \to \mathbb{P}^1$ *by*

$$[a : b : c : d] \mapsto [a : b] \text{ or } [c : d]$$

and $\pi_2 : X \to \mathbb{P}^1$ *by*

$$[a : b : c : d] \mapsto [a : c] \text{ or } [b : d]$$

.

Exercise. *The segre map* $\mathbb{P}^1 \times \mathbb{P}^1 \to \mathbb{P}^3$ *is given by*

$$([x_0 : x_1], [y_0 : y_1]) \mapsto [x_0 y_0 : x_0 y_1 : x_1 y_0 : x_1 y_1].$$

Show the image is $X = \mathbb{V}(ad - bc) \subset \mathbb{P}^3$ *and* π_1, π_2 *are identified with projection maps.*

Exercise. *Show that a regular map* $\mathbb{P}_k^n \times \mathbb{P}_k^m \to \mathbb{P}^N$ *is locally of the form*

$$([x_0 : ... : x_n], [y_0 : ... : y_m]) \mapsto [g_o : .. : g_N]$$

where $g_i \in k[x_0, ..., x_n, y_0, ..., y_m]$ *are bihomogeneous functions of the same degree.*